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Fractal Structure of Molecular Clouds

Srabani Datta

Department of Applied Mathematics, Science College,
University of Calcutta, 92 A.P.C. Road, Calcutta 700 009, India

Abstract.
Compelling evidence exists to show that the structure of molecular clouds is fractal in nature. In this poster, the author reiterates that view and, in addition, asserts that not only is cloud geometry fractal, but that they also have a common characteristic - they are similar in shape to the Horsehead in Orion. This shape can be described by the Julia function f(z) = z^2 + c,where both z and c are complex quantities and c = -0.745429 + 0.113008i. The dynamical processes responsible for the production of these clouds seem to be turbulence followed by Brownian motion till high densities are reached, at which point structure formation is dictated by gravity. The author presents image analysis of four varied examples, namely those of the Horsehead nebula, Helix nebula,Eagle nebula, Rosette nebula and Paley I nebula to prove her hypothesis. The images of fractal these nebulae are analyzed for their box dimension using fractal analysis software and comparisons are made with the given Julia set.

Key words: Molecular clouds, fractal , dimension

1. Introduction

Molecular clouds are self-gravitating, magnetized, turbulent, compressible fluids (J.P.Williams et.al. 1999; Elmegreen 1999; R.B.Larson 1995). In addition, their molecular structure is similar over a wide range of scales with similar power law indices, quite independant of the star-forming nature of the cloud, even at a resolution of a few parsecs. The only difference between these clouds has been found to be in their average densities in that in regions of high densities, gravity begins to dominate and self-similarity breaks down, ultimately leading to the formation of stars, planets, etc. This implies a hierarchy of structure, beginning from the giant molecular clouds with sizes about 50 pc

and masses ranging from 10^4 to 10^6 $M_\odot$.The next class is the normal clouds with sizes ranging from a pc to 30 pc and masses from 25 $M_\odot$ to 10^4 $M_\odot$ (S.S.Prasad 1995). These clouds may or may not be bound and may contain a small number of low mass stars. However, their contribution to the total star formation rate in the Galaxy is negligible. The third is the clump within the cloud. Clumps have sizes ranging from 0. 4 to a few pc amd masses from a few to 25 $M_\odot$. They are coherent star-forming regions out of which stellar clusters form. The gas clumps are normally bound by pressure rather than by gravity. However, both clumps bound by strong self-gravity and by external pressure have identical density distributions. The fourth and final class is that of cores found within clumps, which have sizes 0.1 pc tp 0.4 pc and masses less than 1 $M_\odot$. Cores are regions out of which single stars (or multiple systems) are born and are always gravitationally bound. Stars formed in cores may accrete matter from the host clump or cloud at the protostar stage.

Power-law relationships in molecular clouds have been found in earlier quantitative studies (Kramer et. al.1998; Hetem & Lepine 1993;Stutzki et.al.1998). In this poster, the author investigates the structure of certain molecular cloud images, by using a fractal dimension analysis software. The images presented are of (i) Horsehead nebula (B33) in Orion (Kramer et. al. 1996), (ii)Eagle nebula (M16) in Serpens (Hester et.al. 1996), (iii)Rosette nebula (NGC 2237 to NGC 2246) (Carlqvist et.al. 1998;Clayton et.al. 1998; White 1997) and (iv) IRAS 02356-2959 (Paley I) in Fornax (Paley et.al. 1991; Stark 1995).Of these four, that of the Horsehead nebula has been obtained from the author's observations from the 2.3m Vainu Bappu Telescope, Kavalur, India. Basic mathematical background for this work is presented in section 2. , observations in section 3., analysis in section 4. Finally, discussions are presented in section 5.

2. Mathematical Background

Fractals are self-similar objects, having fine structure. Examples of fractal objects include the von Koch curve, the Sierpinski gasket, the Cantor set and the Julia set(K. Falconer 1997; H.O. Peitgen et.al. 1992). In order to study fractal structure in physical objects, we need a quantitative measure defined on them. Such a quantity is the principal definition of fractal dimension , other definitions, such as Kolmogorov (or Minkowski) dimension and box (or grid) dimension, are widely used. Indeed, in practise, box dimension is the easiest to work with and is equivlent to Kolmogorov dimension.The box-counting dimension is defined in the following way : if X is any non-empty bounded subset of R^n and $N_\epsilon(X)$ be the smallest number of sets of diameter at most ϵ which can cover X, then box dimension is defined as

$$dimX = \lim_{\epsilon \to 0} logN_\epsilon(X)/log(1/\epsilon)$$

In the above definition of dimension, the fundamental assumption is that for each ϵ, we measure a set in such a manner that irregularities of size less than

ϵ will be ignored. For example, if X is a plane curve, then $N_\epsilon(X)$ will be the number of steps required by a pair of dividers set at length ϵ to traverse X. We have to also keep in mind that for real phenomenon we can use only a finite range of ϵ. An example of a fractal object is that of the Julia set given by the function

$$f(z) = z^2 + c$$

where z and c are complex numbers and c = -0.745429+0.113008i(Fig.1) However, unlike Euclidean objects, a fractal object retains a similar level of complexity for each value of ϵ, the fractal dimension gives an index of the degree to which a fractal structure fills the space in which it is embedded (B.Mandelbrot 1982) Hence, examination of an object for self-similarity at different scales of magnification can suggest whether the object has a fractal structure.

3.Observations

Images of the Horsehead nebula were obtained with the 2.3 m Cassegranian telescope at the Vainu Bappu Observatory (VBO) located in Kavalur, India, 1^{st} and 3^{rd} April 2000. The SITe 1024x1024 pixels, 24 micron charge-coupled device (CCD) was placed at it's prime focus. The camera projects each pixel to 0.66" on the sky (resolution of 27.6" per mm) giving a field of view of 11.264 x 11.264 ". The images were obtained through H-alpha (6563 Å)(Fig.2), V and R filter with FWHM of H-alpha filter being 100 Å, while the V and R filters were Bessel filters with the standard bandpass. The effective exposures of the images are given in Table 1. The seeing during the observation of the order of a few arc seconds. Each image was bias subtracted and flat-fielded using twilight flats obtained each day. However, dark currents were assumed to be neglible as the camera was cooled to about - 90^0 C. Both bad regions and cosmic rays were removed using IRAF NOAO image reduction software. The CCD, with a read-out noise of the order of 8 electrons, was calibrated using IRAF software. The standard stars used for guiding the telescope were obtained from the Hubble Space Telescope (HST) Guide Star Catalogue.

Table 1. Observation details from the 2.3m VBT, Kavalur.
Co-ordinates are based on year 2000.

Date	Right Acsension	Declination	Filter	UT	Exposure time
2.4.00	5h40m58.6s	$-2^0 27' 23.84"$	H-alpha	1357	30
3.4.00	5h40m58.6s	$-2^0 27' 23.84"$	R	1353	15
3.4.00	5h40m58.6s	$-2^0 27' 23.84"$	V	1426	30

4. Fractal Analysis

The images (Figures 3 to 5) were obtained from various sources (HST website; D.Malin 1983;R.Stark 1995) and converted to Windows bitmaps. This causes the digitized image to be stored as a data matrix where pixels belonging to the pattern are stored as 1 and pixels from the background are stored as 0, or vice versa. This procedure can also be carried out for colour images (which are overlays of 3 grey-scale images. The fractal dimension of these images were then measured using an implementation of the box-counting method.The automated fractal image analysis software, Benoit 1.3(Truesoft International Inc.,St.Petersburg, USA), was used to convert the solid images to black and white pixel outlines.This software has been reviewed(Science 1999,285,pg.1228)and has been found to perform satisfactorily. These images were overlayed with grids in such a way that the minimum number of boxes were occupied.The is accomplished in Benoit by rotating the grid for each size through 90 degrees at various angular increments of rotation.The user is able to change the box length decrease factor as well as the size of the largest box.

Figure1.The Julia set for c = -0.745429+ 0.113008i(Box dimension=1.679594)

For the analysis, a set of ten measurements were taken at fixed grid rotation increments of 15 degrees. An average dimension was calculated and the number of iterations giving the slope with the least standard deviation (SD) while being nearest to the to the average was chosen. Log-log graphs were then plotted of

Figure2. H-alpha image of the Horsehead nebula obtained with the 2.3m VBT, Kavalur(Box dimension=1.6965725)

Figure 3. Part of PaleyI (Stark 1995)(Box dimension= 1.67718)

Figure 4. Closeup of a trunk of the Rosette nebula (Malin 1983)(Box dimension=1.78261).

the reciprocal of the side length of the square against the number of outline containing squares. The slope is estimated by fitting a line using the method of least squares.Ten further sets of measurements were taken with this selected number of iterations and box size decrease factor. Results for each cloud were tabulated and then the Student's t test of significance (Table 2) was used to test for their deviation from the value of the dimension for a Euclidean shape(topological dimension 1). A further test was made for fluctuations of the cloud dimension values from that of the Julia set. The results show that the cloud dimensions are significantly different from it's topological dimension at 1

Figure 5. Closeup of a trunk of M16(part of column I in Hester 1995)obtained from the Hubble Picture Gallery(dimension=1.669622)

percent cofidence limit (t value 3.25) but not from the Kolmogorov dimension of the Julia set. Hence, it can be said with certainity that molecular clouds are not only fractal but also that their dimension is that of the given Julia set .

Table 2. Analysis of box dimension .

Name	Pattern size(pixels)	Dimension	Student's t value
Julia set	794x1324	1.679594	91.19396
Horsehead nebula	406x473	1.6965725	16.52918
Paley I	463x442	1.677181	95.62187
Rosette nebula(close-up)	109x81	1.78261	85.69697
Eagle nebula(close-up)	188x212	1.669622	70.454

5. Discussion

The discovery of power-law relationships in molecular clouds by Larson(1981) was a ground- breaking study in favour of self-similarity, especially so since a power-law can only be applied to clouds with masses greater than 10^4 $M_\odot$ and does not apply to cores and gravitationally bound clumps. My study on the different types of nebulae reinforces this view, especially since the parts studied are small sections of the main trunk in some clouds (eg. Eagle and Rosette). What is significant here is that their dimension is the dimension of the Julia set. This result implies that physical features of clouds can be explained if we make the following assumptions(i) cloud structure resembles the Julia set corresponding to the Mandelbrot constant given by c = -0.745429+0.113008i and that it is a dynamical entity composed of cloud matter, (ii) growth occurs around the central point (an attractor) of each spiral making it a potential site of star formation, (iii) groups of such central points spiral inwards towards a massive central point, identifying such central points to be dark objects. Such central objects may become supermassive black holes (Macchetto et. al. 1997) or remain as numerous small objects(Goodman and Lee 1989), perhaps

even composed of strange matter (van der Marel 1997; Weber 2000; Chandra & Goyal 2000) (iv) central points increase in size as we move away from the massive central attractor, (v) growth also occurs along each spiral of the set in the form of horseheads, whose sizes reduce adinfinitum as the spiral moves towards the attractive fixed point and finally (iv) after the elapse of sufficient time, when adequate matter has accumulated in the centre, a dwarf, a star or group of stars would form, depending on the size of the clump.

References

Carlqvist, P., et. al., 1998, Astron. Astrophys. 332, L5.
Chandra, D. & Goyal, A.,2000, Phys. Rev. D, in press.
Clayton, A., et. al., 1999, Astron. Astrophys. 334, 264.
Elmegreen, B.G., 1999, Ap.J 527, 266.
Falconer,K.,1997,Fractal Geometry,Wiley and Sons, Chichester .
Goodman,J. and Lee, H.M. ,1989, Ap.J. 337, 84.
Hester, J.J., et.al., 1996, A.J. 111, 2349.
Hetem, A.jr. and Lepine, J.R.D., 1993,Astron. Astrophys.270, 451.
Kramer, C.; Stutzki,J.and Winnewisser,G.; 1996; Astron.Astrophs. 332,264.
Kramer,C.et. al.,1998, Astron. Astrophys.329, 249.
Larson, R.B.,1981,MNRAS 194,809.
Larson, R.B., 1995 ,MNRAS 272, 213.
Malin, D., 1983, A View of the Universe, CUP.,pg.133.
Mandelbrot, B.B., 1982, The Fractal Geometry of Nature, Freeman, N.Y.
Larson, R.B., 1995 ,MNRAS 272, 213.
Paley, E., et.al.,1991, Ap.J. 376, 335.
Peitgen, H.O., Jurgens, H. and Saupe, D., 1992, Fractals for the Classroom- Introduction to Fractals and Chaos, Springer-Verlag, N.Y.
Prasad, S.S. ,1995, Fragmentation of Molecular Clouds and Star Formation, eds.E. Falgarone, F. Boulanger and G. Douvert, Kluwer Academic, Dordecht, p. 93.
Silk, J., 1995, Ap. J. 438, L41.
Stark, R., 1995, Astron. Astrophys. 301, 873.
Stutzki, J., et. al., 1998, Astron. Astrophys. 336, 697.
van der Marel,R.P., et.al.,1997, Nature 385, 610.
Weber, F., 2000, astro-ph 0008376.
Williams, J.P., et. al., 1999, astro-ph 9902246.
White, G.J., et. al., 1997, Astron. Astrophys. 323, 931.

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